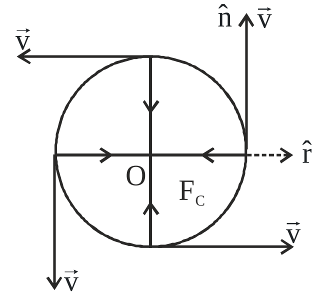
CIRCULAR MOTION

Motion along a circular path : when a body is moving along a circular path with constant speed called uniform circular motion, when speed is not constant, motion is said to be non-uniform circular motion.

$$\vec{F}_c = \frac{mv^2}{r}(-\hat{r})\ ;\ \hat{r} = \text{unit vector along radially outward}$$

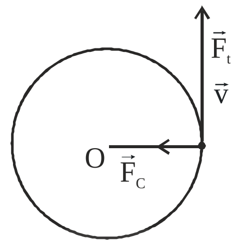


A force required to keep of body on circular path always acts in radially inward direction called centripetal force whose magnitude is $\frac{mv^2}{r}$.

For non-uniform circular motion

$$\vec{F} = F_c(-\hat{r}) + F_t(\hat{n})$$

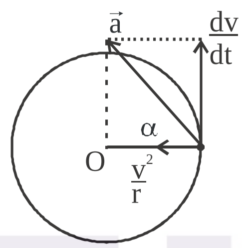
$\hat{n}$ = unit vector along direction of motion or velocity



$$F_c = \frac{mv^2}{r} \text{ and } F_t = m.\frac{dv}{dt}$$

$$\vec{a} = \text{Resultant or net acceleration} = a_c(-\hat{r}) + a_t(\hat{n})$$

$$= \frac{v^2}{r}(-\hat{r}) + \frac{dv}{dt}(\hat{n})$$



$$|\vec{a}| = \sqrt{\left(\frac{v^2}{r}\right)^2 + \left(\frac{dv}{dt}\right)^2}$$

$$\alpha = \tan^{-1}\left(\frac{dv/dt}{v^2/r}\right)$$

Angle of banking: Angle by which an outer edge of circular track is raised to provide the necessary centripetal force through the horizontal component of normal reaction.

angle of banking $\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$

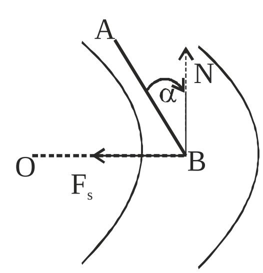
Motion of vehicle on a horizontal circular track:

$\frac{mv^2}{r}$ is being provided by force of static friction i.e., $F_s = u_s N$ and N=mg $\Rightarrow v^2 = u_s rg$ or $v = \sqrt{u_s rg}$

Condition for no skidding on circular track

$$F_s \ge \frac{mv^2}{r} \text{ or } u_s mg \ge \frac{mv^2}{r} \text{ or } v \le \sqrt{u_s rg}$$

Angle of bending of a cyclist on a rough horizontal circular track to move on is given

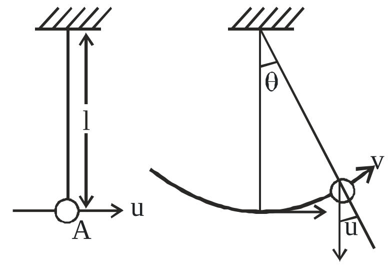


$$\tan\theta = \frac{v^2}{rg} \Rightarrow \theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$

F_s provides necessary centripetal force $\frac{mv^2}{r}$ and N=mg. For safe turn there is a rotational equilibrium hence no torque about A (Centre of gravity of cycle and cyclist).

Vertical Cricular Motion

u is the velocity imparted at the bottom of the vertical circle. At P, equation of motion



$$T - mg\cos\theta = \frac{mv^2}{r} \quad ...(i)$$

and from mechanical energy conservation principle,

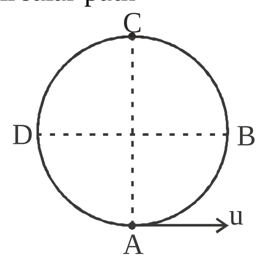
$$\frac{1}{2}mu^2 = mgl(1-\cos\theta) \Rightarrow v^2 = u^2 - 2gl(1-\cos\theta) \quad ...(ii)$$

from (i) and (ii) $T = mg\cos\theta + \frac{m}{l}\left[u^2 - 2gl(1-\cos\theta)\right]$

$$= \frac{m}{l}\left[u^2 - 2gl + 3gl\cos\theta\right] \quad ...(iii)$$

from (ii) and (iii) we have velocity and tension at any point on the verticle circular path

For just to complete the verticle circle



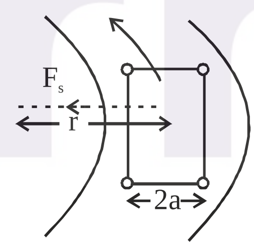
$u = \sqrt{5gl} = \text{velocity}$

At A, $v_A = \sqrt{5gl}$; $T_A = 6mg$ = tension in string when block is at A

At B, $V_B = \sqrt{3gl}$; T_B = 3 mg

At C, $V_C = \sqrt{gl}$; T_C = mg

For no toppling of the automobile on horizontal circular track



$F_s.h \le mga$;

$\frac{mv^2}{r}.h \le mga$

h is the height of centre of gravity of automobile from surface of road.

$v \le \sqrt{\frac{arg}{h}}$

While toppding wheels nearer to centre of track loose the contact.